May 2016 **Sample Midterm 4** **Name:**

Math 250A

1. Let $\mathbf{F}(x, y) = (2 + y + e^x \sin y)\mathbf{i} + (4 + x + e^x \cos y)\mathbf{j}$. Find a function f such that $\mathbf{F} = \nabla f$.

2. Evaluate $\int_C (\sin(\sqrt[3]{x}))dx + (x^3 + 3xy^2)dy$, where C is positively oriented and is the boundary of the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

3. Find the work done by the force field $\mathbf{F}(x, y) =< xy^2, x + y >$ on a particle that moves along the line segment from (2, 4) to (2, 1) : (Write in parametric form)

4. Let C be the portion of the parabola $y = x^2$ where $0 \leq x \leq 1$. Evaluate

$$\int_C \frac{y^2}{x^3} ds$$

5. Let $f(x, y) = xy\cos(y)$ and $\mathbf{F}(x, y) =< y\cos(y), x\cos(y) - xy\sin(y) >$: It can be shown that $\mathbf{F} = \nabla f$ (do not show this): Use this to evaluate $\int_C \mathbf{F.\ dr}$, where C is the curve $r(t) =< t^2, t >; 0 \leq t \leq \pi$

6. Use Green's Theorem to evaluate $\int_C F.dr$; where $F(x; y) =< e^{2x} + x^2y, e^{2y} - xy^2 >$ and C is the circle $x^2 + y^2 = 1$ oriented clockwise.

7. Calculate the mass of the quarter of the unit sphere lying in the first octant if it has a density function of $\delta(x, y, z) = x^2 + y^2 + z^2$.

8. Let $\mathbf{F}(x, y, z) =< \sin(xy), y + z, x - yz >$
   (a) Calculate div$\mathbf{F}$.
   (b) Calculate curl $\mathbf{F}$.

9. Let $f(x, y) = x^2y$ and let C be the top half of the circle of radius 3, centered at the origin, oriented counterclockwise. Evaluate

$$\int_C f(x, y)\ ds$$

10. Let $\mathbf{F}(x, y) =< 1 + y, x - \sqrt{y} >$
    (a) Verify that $\mathbf{F}$ is conservative.
    (b) Find a potential function f for $\mathbf{F}$.

11. The cylinder $x^2 + y^2 = 4$ can be parameterized as follows: $\mathbf{r}(u, v) =< 2\cos u, 2\sin u, v >$. Use this parametrization and an appropriate double integral to calculate the surface area of the portion of this cylinder that lies above the xy-plane and below the plane z = 4.

12. Set up an iterated integral that would give the flux of $\mathbf{F}(x, y, z) = xze^y\mathbf{i} - xze^y\mathbf{j} + z\mathbf{k}$ across the surface S which is the part of the plane $x + y + z = 9$ in the first octant and has upward orientation. (Simplify, but do NOT evaluate)